E 2351. *Proposed by Stefan Porubsky, Comenius' University Bratislava, Czechoslovakia*

Let $\phi$ denote Euler's totient function and let $\tau(n)$ denote the number of divisors of $n$. Show that

$$\phi(n)\,[\tau(n)]^2 \leqq n^2$$

for all positive integers $n \neq 4$. For what $n$ does equality hold?

E 2352. *Proposed by Marlow Sholander, Case Western Reserve University*

For each positive integer $n$, define

$$Q_n = \left[1 + \frac{1}{n}\right]^{n^2} \frac{n!}{n^n \sqrt{n}}.$$

Show that the sequence $\{Q_n\}$ is monotonely decreasing and find its limit.

E 2353. *Proposed by J. G. Rau, Litton Systems, Culver City, Cal.*

Given two sequences $\{a_1, a_2, \cdots, a_n\}$ and $\{b_1, b_2, \cdots, b_n\}$ of positive real numbers, find the permutation $(j_1, \cdots, j_n)$ of the integers $1, 2, \cdots, n$ for which

$$\sum_{m=1}^{n} \sum_{k=1}^{m} b_{j_m} a_{j_k}$$

is a minimum.

E 2354. *Proposed by L. Carlitz and R. A. Scoville, Duke University*

Let $S = \{1, 2, \cdots, n\}$ and let $D_n$ denote the number of permutations of $S$ with no fixed points (derangements). Let $E_n$ denote the number of even permutations of $S$ with no fixed points. Show that

$$E_n = \binom{n}{2} D_{n-2} - (-1)^n (n-1), \qquad n = 2, 3, \cdots.$$