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SEPTUAGENARIAN OTO STRAUCH

On May 15, 2013 entered our friend and esteemed colleague
Oto Strauch the circle of septuagenarians.

Oto traditionally has not done much for birthdays. Moreover, once one reaches a certain age he/she starts hoping the friends will not notice when the odometer registers another year. But this time it is different, to celebrate round 70 in full strength, full of energy and with unceasingly sparking new ideas, makes this occasion unique and unrepeatable.

Oto Strauch was born in Pezinok, a small town near Bratislava, where his parents were evacuated from Bratislava during the war events.[1] His father was an officer on the Ministry of railways, and his mother was an educator in primary schools.

After World War II they returned back to Bratislava, the city, which he remained loyal all his life. After finishing his basic school attendance, he firstly went a non-mathematical way. He finished with a matura a 4-year chemical technical secondary school in 1962. Then he inscribed to the Comenius' university in Bratislava. At that time it was not possible to study only the mathematics from the very beginning. One could start to study mathematics only in classes together with future secondary school teachers, who studied mathematics in combination with some other subject, as physics, descriptive geometry, geography, etc. Then at the end of the fourth semester it was possible to switch to pure mathematics, provided your hitherto study results were evaluated by your educators as satisfactory for such a change. Moreover, the class of pure mathematics was very tiny at that time at the university in Bratislava, on an average so about 3–5 students. There was also possible to continue after the fourth semester in one subject study oriented in numerical mathematics (approximately 10–15 students) or statistics (approximately five students). Accidentally, since the school year 1968/69 on an initiative of Prof. A. Kotzig [ARST] a new course in econometric was opened (approximately 10 students).

Oto decided for mathematics in combination with physics. The physics, however, contained seminars of tedious experiments in physical labs, which were connected with at least half-days measurements and the subsequent data-processing of the obtained measurement results (consisting of composition of party pointless and absurd protocols or surveys, to say truth) every week. For some fun, for someone two years of suffering. Oto belonged to the second category. He found this part of the study having no influence on one's mathematical growth as a pure time wasting, and so he decided to neglect them as much as possible in his curriculum. At the beginning the physicists turned a blind eye to his absence on labs classes during the first two semesters and so to prevent his definite exclusion from the study at its beginning. However, during the second

[1]It is interesting to note that most aerial battles between the Russian and German armies took place just over the region of the Little Carpathians around Pezinok.

year the physicists decided not to tolerate more his absence on labs classes, and so Oto was forced to leave up his first attempt to be a mathematician. In the former communist regime early termination of the study resulted in an immediate call to a compulsory military service in duration of two years. After finishing the service Oto entered the university again in 1966, and despite his previous experience with physics, again as a student of the combination mathematics with physics. Though the military service experience leaves unavoidable traces on one's forms and ways of perception and coexistence with the surrounding world, the problems with the physical labs continued. This time, however, he succeeded to overcame all the pitfalls and obstacles connected with the unloved subjects and so we find him in the fifth semester in the class of pure mathematics with three other fellow students (Štefan Porubský, Peter Mederly and Alois Némethy). Now rose to the surface another malady of the communist education system. Actually, during the first six semesters of the university study the students were obliged to attend lectures on three so-called basic components of the marxism-leninism: history of the international working-class movement, the marxists philosophy, and the marxists economy. Even if it was the time of a political relaxation in the second half of 60's culminating during the so-called Prague spring, it is a little mysterious how Oto, who principally hated the violence and the ruling elite, passed the exams from these three subjects. For instance, at the end of the sixth semester suddenly arose a rush for Oto. One of the authors (Š.P.) of this lines met on the street short time before the end of the exam period and the summer 1969 the late Prof.M.Kolibiar [Ka95] asking for Oto with the comment, that he has a message that Oto did not yet made the exam from the marxists economy. Without passing this exam it was not possible to continue the study. Nevertheless, the story had again a happy end, and Oto came to the matriculation in September 1969, and since then in the following four semesters he could fully concentrate on mathematics only. He graduated in mathematics in 1971.

A characteristic phenomenon of that student times was the lack of the literature. Although good books in the Russian language existed, it was not possible to buy them as necessary due to inability of the former market mechanism to handle demands. It was possible to buy them only shortly around the time of their publication, and to buy a particular book later, say one year, was like a windfall. The English mathematical books were expensive and their import was limited, though they were practically politically unobjectionable. So the only source of knowledge were one own's (or friend's) notes from lectures, or books in public libraries. From this reason, if the need arose to meet Oto, one could be almost sure to find him in one of the three public libraries in the city center (and not far from each other) which he regularly visited, and where he spent his out-of-school time (it is a certain irony that one of them was the reading room of the library of the marxism-leninism).

A very popular possibility of that time to widen one's knowledge and mathematical experience was to participate on the work of many mathematical seminars. There were two types of seminars, ones as a part of the curriculum, and then seminars which were organized within mathematical departments, and which took place out of the classes. They gathered mathematicians of all generations interested in the subject, and often outside the city. However, Oto as a confident loner (but a loyal and reliable friend) did not join such seminars, and relied on its own initiative and sense. In his free time he studied theories of his own choice, and these were as we know him today, the number theory, the classical analysis, the theory of real functions, measure theory, etc. In the fourth year of the study it came the time to select the topic of the diploma thesis. Lists with theses themes proposals and contact information of prospective supervisors hang in the premises of each department. In the case of Oto it was the other way around, he contacted his supervisor with the already almost finished theses. This supervisor was Professor T. Neubrunn [RS91], who after reading the manuscript agreed to assume the role of the supervisor and so Oto successfully defended his diploma thesis *Set systems closed with respect to certain set operations* in 1971. Professor Neubrunn was a close friend and collaborator of another significant figure of the mathematical life in Bratislava – Professor Tibot Šalát [KS] who headed two mathematical seminars for a wide interested academic community. One was from the theory of real functions, and the second one from the theory of numbers.

The first one, as an actual field of interest of Professor Šalát, had a longer tradition, and as Oto's list of publications shows, it was actually a springboard for his future professional mathematical orientation. Actually both met at the latest in eight semester, on course lectures on constructive theory of real functions and a prescribed class seminar. Professor Šalát's enthusiasm and approach to students attracted the interest even of those students who were not examined by him. Oto joined his seminar from real functions somewhere at the beginning of 70ies, and later he also joined the seminar from number theory. The liaison between both become strong and lasted until Professor Tibor Šalát's death in 2005. Oto wrote his thesis *Minimal cover of the closed interval* for the degree RNDr. (rerum naturalium doctor[2]) under his supervision in 1971 and Professor

[2]In Czechoslovakia, as a one of the successors of the former Austrian-Hungarian Empire the system of degrees originated in the law from April 15, 1872, No. 57/1872 of the Ministry of Culture and Education of the Empire. After creation of Czechoslovak Republic, the Philosophical

Šalát became also the supervisor of his PhD thesis *Metric theory of Diophantine approximations and a connection of diophantine approximation with differentiability of some real functions* which he defeated (after various delays, mainly political) in 1980.

One of the special features of Professor Šalát's seminars was his liking for problems proposed in each issue of the American Mathematical Monthly. He liked to submit them regularly to the participants of both seminars. Oto was not only their ardent solver but also a proposer. His name it is possible to find for the first time among the solvers of the problem E2368 (1973, p. 811–812).

After finishing his study in 1971 Oto remained at the Faculty of Science of the Comenius' University in Bratislava for the period 1971–1980, and after its reorganization in 1980 at the newly established Faculty of Mathematics and Physics till 1986. However the aversion between him and communistically oriented faculties left a trail on his carrier. He was given only short-term contracts on various non-attractive positions for years. Thus in 1971–1973 he was appointed as a programmer – analyst in the Computer Center of the Faculty of Natural Sciences. In 1973–1976 he was Ph.D. student at the Department of Algebra and Number Theory. In 1976–1984 he was a lecturer at the same Department mostly with one year contracts. In 1984–1986 he was moved up as a lecturer to the Department of Applied Mathematics. This all under the condition to administrate various disliked bureaucratic agenda as the scientific secretary at the corresponding departments in years 1982–1986 having with each of them, as mentioned, always only a one-year contract. During this period he led exercises, seminars and lectured on various topics, e.g., on mathematical analysis, number theory, topology, operation research, etc. All this on the background of a constant struggle with mathematically meaningless faculties representing the communist regime. In 1986 he moved finally to Mathematical Institute of the Slovak Academy of Sciences in Bratislava, where he is until today.

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faculties were divided into two parts: the Philosophy and Natural sciences, and so the degree PhDr. still valid for philosophical sciences obtained an analogue RNDr. for natural sciences. This was then changed by the communistic government by the law No. 58 from the year 1950 with the operation from the year 1953. Since the Russian equivalent of the degree PhD was the CSc. (abbreviation for Candidatus Scientiarum following the Russian кандидат наук) the original corresponding degree RNDr. was abolished. It was reintroduced in 1966 in a downgraded form. To get this degree (somewhere between M. A. and PhD) in mathematics it was necessary to have at least one publications in a journal and to pass a committee examination on the level of the final state exam on a university.

In 1997 he defeated here his habilitation theses *Distribution of sequences* on Faculty of Mathematics and Physics of the Comenius' University in Bratislava, and in 2002 his DrSc thesis *Distribution of sequences* in Mathematical Institute.

In 1981–1986 Oto was the managing editor of the journal Acta Mathematica Universitatis Comenianae issued by the mathematical departments of the same university, in 1988–1993 he was managing editor of Mathematica Slovaca issued by the Mathematical Institute of the Slovak Academy of Sciences in Bratislava. Oto is also the founder of the scientific seminar on the probabilistic number theory, which is already more that a decade held at the Mathematical Institute every Tuesday from 10:00 till 12:00. These his activities culminated when in 2006 he succeeded to realize his old dream to have a journal fully devoted to the uniform distribution and related topics, when he founded a new journal Uniform Distribution Theory issued till 2010 by the Mathematical Institute. In 2011 its editorial office moved from the Mathematical Institute to the BOKU-University in Vienna.

On this occasion, it is necessary to mention Oto's careful agenda keeping not only in connection with the editorial agenda. The piles of sheets on his shelves (and elsewhere around) in perfect order and on top of each the exact list of what's underneath located. Everything written by Oto's typical prefigurative handwriting. His orientation in all the piles is fast, reliable and inerrable. The mathematical community knows him also as an author of extremely thoroughly written reviews in Mathematical Reviews or Zentralblatt für Mathematik. Archived handwritten drafts, comments on the reviews can be found again in piles on the shelves together with pointers to the related ones.

Oto is also one of the founders of biennially organized conferences called UDT conferences, which as indicated, are devoted to the subject of the theory uniformly distributed sequences and related topics.

He supervised three PhD students: János T. Tóth (defence 1998), Oľga Blažeková (defence 2008), and Jana Fialová (defence 2013).

Last but not least, Oto is an enthusiastic walker. Bratislava and over it and Danube beginning and to them leaning Little Carpathians forming about 100 km long starting mountain range of the Carpathians arc are an ideal place for picturesque walkabouts. Oto spent countless hours in their womb and knows their every nook, all shortcuts and paths and only hardly anyone can surpass him in the knowledge of their corners. Many of his results found their birth-hour on the pathways in the woods around Bratislava.

Mathematical works

The scientific work of Oto Strauch[3] can be divided on two major parts:

I. Metric theory of diophantine approximation.

II. Theory of uniformly distributed sequences.

Part I: The metric theory include the papers [5–10, 44] devoted the famous Duffin-Scheaffer conjecture:

Let $f(q)$ be a function defined on the positive integers and let $\varphi(q)$ be the Euler totient function. Then the Duffin and Schaeffer conjecture [D.S.C.] claims that given an arbitrary function $f \geq 0$ defined on positive integers (but the zero values are allowed for f) the diophantine inequality

$$\left| x - \frac{p}{q} \right| < f(q), \qquad \gcd(p, q) = 1, \quad q > 0 \tag{1}$$

has infinitely many integer solutions p and q for almost all $x \in [0, 1]$ if and only if the series $\sum_{q=1}^{\infty} \varphi(q) f(q)$ diverges.

The D.S.C. is one of the most important unsolved problems in metric number theory. According to V. G. Sprindžuk [Sp79] the answer may depend upon the Riemann hypothesis.

Strauch wrote an expository entry 2.5 on it in [49] and two entries in the Encyclopaedia of Mathematics [Haz20, pp. 172–174, 242–243] devoted to Duffin-Schaeffer conjecture and Gallagher ergodic theorem. His results from [5–7, 10] are also quoted in the monograph [Ha98] by G. Harman.

A class of sequences q_n, $n = 1, 2, \ldots$, of distinct positive integers and a class of functions $f(q) > 0$ is said to satisfy the D.S.C. if the divergence of

$$\sum_{n=1}^{\infty} \varphi(q_n) f(q_n)$$

implies that for almost all x there exist infinitely many n such that the (Diophantine) inequality

$$\left| x - \frac{p}{q_n} \right| < f(q_n)$$

has an integer solution p that is mutually prime with q_n.

[3] This part is based on his own notes.

The main contribution of Oto Strauch [5] to the conjecture can be reformulated in the following generalized form:

Let x_n be an uniformly distributed sequence in $[0,1]$ such that for infinitely many M there exist

$$c_M,\ c'_M, \quad \text{and} \quad N_0(M) \ \text{with} \ c'_M \to 0 \ \text{as} \ M \to \infty$$

such that

$$\sum_{\substack{|x_i - x_j| \le t \\ M < i \ne j \le N}} 1 \le c_M t (N-M)^2 + c'_M (N-M)$$

for every $N \ge N_0(M)$ and every $t \ge 0$. Then given a non-increasing sequence z_n with $\sum_{n=1}^{\infty} z_n = \infty$, for almost all $x \in [0,1]$, the inequality

$$|x - y_n| < z_n \tag{2}$$

holds for infinitely many n.

This implies among others, that the following sequences q_n satisfy the D.S.C.: $q_n = q^n$, $q_n = n!$, $q_n = 2^{2^n} + 1$ — the sequence of Fermat numbers, $q_n = F_n$ — the sequence of Fibonacci numbers, or $q_n = q^n - 1$.

Part II: The backbone of his scientific work forms his contribution to the theory of uniform distribution, especially to the theory of distribution functions of sequences. As we know, its goal is to characterize properties of the sequence x_n via properties of the set $G(x_n)$ of all its distribution functions.

Let us mention some of Oto Strauch's results:

Result 1. There are open question about distribution of sequence $(3/2)^n \bmod 1$. For instance, it is not known whether $(3/2)^n \bmod 1$ is uniformly distributed or dense in $[0,1]$. In 1968 K. Mahler [Ma68] conjectured that there exists no $\xi > 0$ such that $0 \le \{\xi(3/2)^n\} < 1/2$ for all $n = 0, 1, 2, \ldots$ Mahler's conjecture follows from the following conjecture on distribution functions: If I is a subinterval of $[0,1]$ and there is a distribution function $g(x)$ of $(3/2)^n \bmod 1$ such that $g(x) = \text{constant}$ for all $x \in I$, then the length $|I| < 1/2$.

O. Strauch proved in [22, p. 28] that any distribution function $g(x)$ of $\xi(3/2)^n \bmod 1$ satisfies the functional equation

$$\begin{aligned} g(x/2) &+ g\big((x+1)/2\big) - g(1/2) \\ &= g(x/3) + g\big((x+1)/3\big) + g\big((x+2)/3\big) - g(1/3) - g(2/3). \end{aligned} \tag{3}$$

He also found non-trivial distribution functions satisfying this equation, for instance the piecewise linear function with the following graph (for details consult [48, 2–150]).

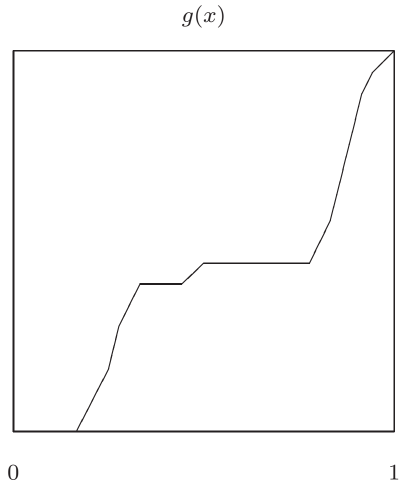

Figure 1.

In [22] he found examples of sets of uniqueness for distribution functions $g(x)$ of $\xi(3/2)^n \bmod 1$, where a set of uniqueness is a subset of $[0,1]$ values over which uniquely determine all of them. For instance, if $I_1 = [0, 1/3]$, $I_2 = [1/3, 2/3]$, $I_3 = [2/3, 1]$, and g_1, g_2 are any two distribution functions satisfying (3) such that $g_1(x) = g_2(x)$ for $x \in I_i \cup I_j$, $1 \le i \ne j \le 3$, then $g_1(x) = g_2(x)$ for all $x \in [0,1]$. These results were extended in [ARS05].

Result 2. In a forty pages long paper [18] he formulated and completely solved a new moment problem inspired by L^2-discrepancy. To a given triple of numbers $(X_1, X_2, X_3) \in [0,1]^3$ find a distribution function $g(x)$ such that

$$(X_1, X_2, X_3) = \left(\int_0^1 g(x) \,\mathrm{d}\, x, \int_0^1 x g(x) \,\mathrm{d}\, x, \int_0^1 g^2(x) \,\mathrm{d}\, x \right).$$

The background of the problem is based on the result: If a sequence (x_n) from $[0,1]$ has a limiting distribution g, then the limits

$$Y_1 = \lim_{N\to\infty} N^{-1} \sum_{n \le N} x_n, \qquad Y_2 = \lim_{N\to\infty} N^{-1} \sum_{n \le N} x_n^2,$$

$$Y_3 = \lim_{N\to\infty} N^{-2} \sum_{m,n \le N} |x_m - x_n|$$

exist and are connected with moments $\int_0^1 g(x) \,\mathrm{d}\, x, \int_0^1 x g(x) \,\mathrm{d}\, x, \int_0^1 g^2(x) \,\mathrm{d}\, x$. In [18] he gave a complete description of those triplets X_1, X_2, X_3 that determine the function g uniquely.

Moreover, he described the boundary of the body

$$\Omega = \left\{ \left(\int_0^1 g(x)\,\mathrm{d}\,x, \int_0^1 xg(x)\,\mathrm{d}\,x, \int_0^1 g^2(x)\,\mathrm{d}\,x \right); g \text{ is a distr. function} \right\}$$

in terms of explicitly defined surfaces $\Pi_1, \dots \Pi_6$ and a curve Π_7 saying that for $(X_1, X_2, X_3) \in \cup_{i=1}^6 \Pi_i$ the moment problem has a unique solution, for $(X_1, X_2, X_3) \in \Pi_7$ exactly two solutions, and in the interior of Ω it has infinitely many solutions (consult [18] or [48, 2–20, 2.2.21] for more details). Moreover, if for a sequence x_n, $n = 1, 2, \dots,$ from $[0, 1]$ the limits Y_1, Y_2, Y_3 exist and we put $X_1 = 1 - Y_1$, $X_2 = \frac{1}{2} - \frac{1}{2}Y_2$ and $X_3 = 1 - Y_1 - \frac{1}{2}Y_3$, then if $(X_1, X_2, X_3) \in \bigcup_{1 \le i \le 7} \Pi_i$, the sequence x_n possess an asymptotic distribution function $g(x)$.

Result 3. In [19] O. Strauch extended the notion of the L^2-discrepancy to an arbitrary sum of the form $\frac{1}{N^2} \sum_{m,n=1}^N F(x_m, x_n)$ where $F(x, y)$ is a real-valued function defined over the Cartesian product of the set of values of x_n with itself. To describe the properties of the underlying sequences x_n, $n = 1, 2, \dots,$ satisfying $\lim_{N \to \infty} \frac{1}{N^2} \sum_{m,n=1}^N F(x_m, x_n) = 0$ he faced the challenge to solve the moment problem

$$\int_0^1 \int_0^1 F(x, y)\,\mathrm{d}\,g(x)\,\mathrm{d}\,g(y) = 0. \tag{4}$$

He proved in [19] that if $G(F)$ is the set of all distribution functions $g(x)$ satisfying (4) and $G(x_n)$ be the set of all distribution functions of x_n, then for a symmetric $F(x, y)$ we have

$$G(x_n) \subset G(F) \iff \lim_{N \to \infty} \frac{1}{N^2} \sum_{m,n=1}^{N} F(x_m, x_n) = 0.$$

In [27] he defined a special co-positive function $F(x, y)$ for which

$$\int_0^1 \int_0^1 F(x, y)\,\mathrm{d}\,g(x)\,\mathrm{d}\,g(y) = \int_\alpha^\beta \big(\mathbf{g}(t) - \mathbf{g}_1(t)\big)\mathbf{A}(t)\big(\mathbf{g}(t) - \mathbf{g}_1(t)\big)^T\,\mathrm{d}\,t,$$

where $\mathbf{g}(t)$ and $\mathbf{g}_1(t)$ are vector functions associated with distribution functions $g(x)$ and $g_1(x)$ and $\mathbf{A}(t)$ is a special associated matrix. If case $\det \mathbf{A}(t) \neq 0$ he obtained a unique solution $g_1(x)$ of (4).

Result 4. From other Strauch's papers, many with co-authors, let us mention some of them very briefly: In papers [12, 17] there are introduced and studied functions $f(x)$ which preserve uniform distribution. In papers [29, 30, 36, 45, 46] distribution functions of block sequences of the type $\left(\frac{x_1}{x_n}, \dots, \frac{x_n}{x_n}\right)$ are studied, where x_n is an increasing sequence of positive integers. Paper [41] is devoted to Benford's law for sequences in $(0,1)$. In papers [32, 33] interesting one-time pad ciphers based on uniformly distributed sequences are proposed and analyzed. In the joint paper [37] properties of the pseudo-random numbers resulting from a quadratic generator $ax^2 + bx + c \pmod{M}$ are studied. Properties of maldistributed sequences are studied in [20, 23], those of sequences connected with the fractional parts of the form $\{n\alpha\}$ in [9, 40], and in [47] complex of question connected with Quasi-Monte Carlo integration in Hilbert spaces of functions are handled.

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