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Ján Plesník; Štefan Porubský; Alexander Rosa; Jozef Širáň
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PROFESSOR ŠTEFAN ZNÁM (1936 — 1993)

Štefan Znám, Professor of Mathematics at Comenius University in Bratislava and a leading Slovak graph theorist and number theorist, passed away suddenly on July 17, 1993.

He was born on February 9, 1936 in Veľký Blh, Slovakia. In 1954 he completed his high school education in Rimavská Sobota. In the same year he started to study mathematics in the Faculty of Science of Comenius University in Bratislava; he graduated in 1959. After spending one year as a secondary school teacher in Piešťany, in 1960 he joined the Department of Mathematics at the Faculty of Chemical Engineering of the Slovak Technical University in Bratislava. In 1967 he took up a position at the Department of Algebra and Number Theory of the Faculty of Science of Comenius University in Bratislava. Later, in 1980, the mathematics and physics departments branched off the Faculty of Science to form a new Faculty of Mathematics and Physics of Comenius University; Štefan Znám worked at the Department of Algebra and Number Theory of this institution until his death. He was promoted to the rank of Associate Professor in 1968 and became a Full Professor in 1982. He completed his graduate studies in 1966 under the supervision of Professor Štefan Schwarz and obtained the degree of Candidate of Science (which roughly corresponds to Ph.D.). In 1980 he received the highest scientific degree DrSc. (Doctor of Science).

Štefan Znám started his scientific career working in number theory. Later in the sixties, partly through working on Zarankiewicz's problem, he became more and more interested in graph theory which was at the time becoming a rapidly expanding discipline.

One of the central themes Štefan Znám's papers in number theory focused on were number-theoretic questions related to graph theory. These papers are typical for combinatorial interests of Znám, where he always oscillated between number and graph theory.

The papers [3], [9], [14], [15] center around the notion of k-thin sets which generalizes the notion of sum-free sets (case $k = 3$ of the following definition). A set $A = \{a_i\}$ of positive integers is called k-thin if there are no solutions of $a_1 + \cdots + a_{k-1} = a_k$ with (possibly equal) elements of A. R. Rado has shown in 1933 that for all $k \geq 3$, $p \geq 1$ there is a largest integer $f(k,p)$ for which the set of all positive integers $\leq f(k,p)$ is a union of p k-thin sets. Generalizing Schur's result for $k = 3$, Znám proved in [9] that $f(k,p+1) \geq k \cdot f(k,p) + k - 2$ for all $k \geq 3$ and p. Moreover, his proof gives a method of partitioning the set of positive integers $\leq \frac{(k-2)(k^{p+1}-2)}{k-1}$ into p k-thin sets. This gives a lower bound for the Ramsey number $R(k,p)$, the smallest positive integer such that if all edges of a complete graph on $R(k,p)$ vertices are colored by p colors, there will be a monochromatic complete subgraph on k vertices.

In [15] he presented an interesting reformulation of the above problem for the case $p = 2$: $R(k,2) = u(k) + 1$, $k \geq 3$, where $u(k)$ is the greatest positive integer such that there exist $u(k)$ positive integers with the property that in an arbitrary k-tuple of them there exists at least one pair of coprime and at least one pair of non-coprime numbers.

In [14] the results of [9] were extended one step further. Let $f(k,n,p)$ be the largest integer N for which the set $\{n, n+1, \dots, N\}$ is a union of p k-thin sets. Znám proved that, for

$k \geq 3$, $f(k,n,p) \geq k \cdot f(k,n,p-1) + (k-n-1)$, and again applied this result to graph colorings.

The impetus for the papers [7], [8] was given by questions connected with cyclic decompositions of complete graphs with $2n+1$ vertices into circuits with n edges. Extracting the number-theoretic background of such decompositions leads to the study of properties of subsets of the set $\{1, 2, \ldots, 2n\}$ satisfying certain congruences. Some interesting connections between such subsets and restricted partition functions are found in these two papers.

Another subject close to Š t e f a n Z n á m's heart was that of disjoint covering systems of residue classes. A system of residue classes

$$a_i \pmod{n_i}, \quad 0 \leq a_i < n_i, \qquad i = 1, 2, \ldots, k, \quad k > 1 \tag{1}$$

is a disjoint covering system (DCS) if every integer is contained in exactly one residue class of the system.

Š t e f a n Z n á m had very close relations to Hungarian mathematicians. Paul Erdős introduced him to the world of covering systems, and it was Erdős who mentioned to him that J. Mycielski made an interesting conjecture on the number of classes in a disjoint covering system. Namely, if $n_{i_0} = \prod\limits_{j=1}^{r} p_j^{\lambda_j}$ is the standard form of a modulus n_{i_0} in (1), then Mycielski's conjecture claimed that $k \geq 1 + \sum\limits_{j=1}^{r} \lambda_j (p_j - 1)$. Z n á m in turn proved this conjecture in [10] before the original conjecture was even published and showed that the bound was best possible. Later he generalized in [20] his proof to show that Mycielski's conjecture is true for systems (1) such that the class $a_{i_0} \pmod{n_{i_0}}$ is disjoint with all the remaining ones, and finally in [34] he showed that for the same conclusion it is enough to require that the class a_{i_0} $\pmod{n_{i_0}}$ contains a positive integer not covered by any other class appearing in the covering system.

His idea was, however, capable of further extension in which the underlying set of positive integers can be replaced by more general algebraic structures. So, for instance, in [28] it was applied to principal ideal domains.

One of the earliest results on disjoint covering systems was a result proved independently by Davenport, Mirsky, Newman, and Rado, according to which in every disjoint covering system there are at least two residue classes with respect to the greatest modulus. S. K. Stein in 1957 showed that if a disjoint covering system contains one pair of residue classes with respect to the greatest modulus and the moduli of the remaining classes are distinct, then all the moduli of such disjoint covering system are uniquely determined. In [21] Š t e f a n Z n á m extended this result to disjoint covering systems with one triple of classes with respect to the greatest modulus and with distinct moduli of the remaining classes. The idea of the proof of this result led Z n á m to make the following conjecture: *Each disjoint covering system contains at least p residue classes with respect to the greatest modulus where p is the smallest prime divisor of the greatest modulus in a disjoint covering system.* Unfortunately, his first proof of this conjecture in [17] was incorrect. However, his improper use of Schoenberg's 1964 result in decomposition of rational polygons led Z n á m [32] to the definition of a new type of disjoint covering systems, the so-called vector covering systems. Given a vector $\varepsilon = (v_1, \ldots, v_k)$ with real components v_i, the system (1) is called ε-covering (or vector covering) if for each integer n we have $\sum\limits_{i=1}^{k} v_i \chi_i(n) = 1$, where χ_i is the characteristic function of the i-th residue class. In [32] some basic properties of disjoint covering systems are proved to be valid also for vector covering systems. In [43] vector covering systems with only one triple or two pairs of classes with respect to equal moduli (and the remaining being distinct) are investigated.

A correct proof of the above conjecture was given in [31] by a new powerful analytical technique which was developed in this paper. Using this technique one can prove again a number of basic results on disjoint covering systems.

The interested reader can consult [41] for more details about results related to Z n á m 's papers on covering systems.

The remaining papers of Š t e f a n Z n á m in number theory have a strong combinatorial flavour as well. Let us mention two of them.

A sequence $\{f_i\}_{i=1}^{\infty}$ of positive integers is said to be complete if every integer (or sometimes only every sufficiently large integer) n can be represented in the form $n = \sum_{i=1}^{\infty} \varepsilon_i f_i$, where each ε_i is zero or one. This notion generalizes a well-known property of Fibonacci numbers. In [13] the following result was proved as a variation on the notion of completeness: Let a positive integer k and a sequence p_i of positive integers be given. Then there exists a unique sequence m_i of positive integers such that every nonnegative integer n can be written in the form $n = \sum_{i=1}^{\infty} a_i m_i$ with $0 \le a_i \le p_i$ and $\sum_{i=1}^{\infty} a_i \le k$.

The well known Syracuse-Kakutani-Collatz conjecture says that if

$$f(n) = \begin{cases} 3n+1 & \text{if } n \text{ is odd}, \\ \dfrac{n}{2} & \text{if } n \text{ is even}, \end{cases}$$

then for every positive integer n there exists i such that $f^i(n) = 1$, where f^i means the i-th iterate of f. The main result of [45] states that it is enough to verify this conjecture for positive integers belonging to any of the residue classes of the type $a \pmod{p^k}$, where p is an odd prime such that 2 is a primitive root modulo p^2 and a is a positive integer coprime to p. Thus the problem is reduced to that over a set of arbitrarily small asymptotic density.

As for graph theory, one of his most important papers is [12]. In this pioneering work in graph decompositions, the authors study the decomposition of a complete graph into factors with given diameters. The basic and beautiful fact is that if the complete graph K_n of order n is decomposable into k factors with diameters $d_1, d_2, \dots, d_k$, then so is any complete graph of order $N > n$ (this is the so-called hereditary property of decompositions). The case $k = 2$ was settled completely, but several interesting results were obtained in the general case. This paper opened a new perspective in graph decompositions and has stimulated a fruitful research by many other authors. By our count, at least 42 successor papers have appeared. For example, it is shown in [12] that K_{11} is not decomposable into three factors of diameter 2 but K_{13} is; whether K_{12} is as well remained a hard open problem for more than 25 years. Only very recently it has been shown, with the aid of computer, that the answer is negative.

Papers [38], [40] deal with the case $d_1 = d_2 = \cdots = d_k = 2$. Let $f(k)$ be the smallest integer such that $K_{f(k)}$ can be decomposed into k factors of diameter two. In [38] Z n á m proved that $f(k) \ge 6k - 7$ for $k \ge 664$. In this connection J. Bosák (1974) and B. Bollobás (1978) conjectured that for sufficiently large k, $f(k) = 6k$. Š t e f a n Z n á m in [40] settled this conjecture in the affirmative.

In [25], analogous questions are studied for complete digraphs. Here the hereditary property was established for two factors. Very surprisingly, the hereditary property is no longer true in general for any even number of factors greater than two. In the case of an odd number of factors the question remains open. Decompositions into factors with given radii are studied in [29].

In his book, O. Ore in 1962 introduced the concept of a geodetic graph as a graph where any two vertices are connected by a unique shortest path. In [16], [19], a subclass of this class was studied. A graph G is strongly geodetic if any two vertices of G are connected by at most one path of length not exceeding the diameter of G. The main result of [19] says that a strongly geodetic graph is either a forest or a regular graph of finite diameter. A beautiful connection to Moore graphs is established in that a finite graph G is a Moore graph if and only if G is a regular strongly geodetic graph.

In [36], Š. Znám introduced the concept of a t-graph as a graph in which every pair of distinct vertices x and y are connected by t paths of length equal to the distance from x to y. He showed that any t-graph of girth greater than the diameter d is necessarily regular.

Analogously, one can define strongly geodetic digraphs. In [30] it is proved that all finite strongly geodetic digraphs are either complete digraphs or cycles. The "nearly Moore digraphs" (with defect 1) were studied in [51], [52].

Štefan Znám also contributed significantly to the development of extremal graph theory, especially by investigating the size of graphs with given diameter and maximum degree. Let $F(n,k)$ be the minimum number of edges of a graph with n vertices, diameter 2, and maximum degree k; this function was introduced by Erdős, and Rényi in 1962. The fact that $F(n,k) = 2n-4$ in the range $(2n-2)/3 \le k \le n-5$ was proved by Erdős, Rényi and Sós in 1966. In [42], sharp results on $F(n,k)$ were proved in the range $(3n-5)/5 \le k \le (2n-3)/3$ for $n \ge 241$. Further results on this topic are in [44], where the following is shown: If $0 < h < 1/14$, $n > 21/h^2$, and $3n/7 \le k \le (0.5-h)n$, then $F(n,k) = 3n-12$. Š. Znám also investigated the asymptotic behaviour of $F(n,k)$ by means of the function

$$f(p) = \lim_{n\to\infty} \frac{F(n, \lfloor pn \rfloor)}{n}, \qquad 0 < p < 1.$$

In [47] he proved that $f(p) = 5 - 4p$ if $5/12 < p < 3/7$. This result was extended in [48] by showing that $f(p) = 8 - 11p$ for $2/5 < p < 5/12$. More results in this area can be found in [26], [39]. A further contribution to metric graph theory is in [46].

Packing problems of a somewhat different type were tackled in [50], [49]. If triangles (or pentagons) are packed edge-disjointly in the complete graph until no triangle (pentagon) is left, a maximal partial triple system (pentagon system) results. The paper [50] completes the determination of the spectrum for possible number of triangles in a maximal partial triple system, while in [49] both the maximum and the minimum number of pentagons in a maximal partial pentagon system are determined. These results rest on a characterization of extremal triangle-free (pentagon-free) bipartite and non-bipartite eulerian and anti-eulerian graphs.

Štefan Znám was very good at attracting young talented students to graph theory and number theory. He supervised more than 15 graduate students. Several of them went to distinguish themselves and are today well-respected mathematicians.

Besides university affairs, Š. Znám was also interested in problems of mathematical education in basic and secondary school (see the books [80], [81], or papers [53], [56]). He was very successful in popularizing mathematics in general, and number theory and graph theory in particular. He wrote more than 20 popular articles on mathematics as well as 5 books and several texts for students. He prepared several TV shows popularizing mathematics and mathematical education.

When Anton Kotzig left for Canada in 1969, Štefan Znám together with Juraj Bosák took over the leadership of the Bratislava graph theory seminar. Š. Znám continued to lead

[38] *Decomposition of complete graphs into factors with diameter two*, Math. Slovaca **30** (1980), 373–378.

[39] *On two problems from extremal graph theory.* In: Colloq. Math. Soc. J. Bólyai 37. Finite and Infinite Sets. Eger 1981, pp. 867–869.

[40] *On a conjecture of Bollobás and Bosák*, J. Graph Theory **6** (1982), 139–146.

[41] *A survey of covering systems of congruences*, Acta Math. Univ. Comen. **40-41** (1982), 59–79.

[42] *Minimal graphs of diameter two and given maximal degree* (with I. Vrťo), Studia Sci. Math. Hung. **17** (1982), 283–290.

[43] *Vector-covering systems with a single triple of equal moduli*, Czechoslovak Math. J. **34** (1984), 343–348.

[44] *Minimal graphs of diameter two*, Studia Sci. Math. Hung. **19** (1984), 187–191.

[45] *A note on $3x + 1$ problem* (with I. Korec), Amer. Math. Monthly **94** (1987), 771–772.

[46] *On equality of edge-connectivity and minimum degree of a graph* (with J. Plesník), Arch. Math. (Brno) **25** (1989), 19–26.

[47] *Minimal size of graphs with diameter two and given maximal degree*, Ars Combin. **28** (1989), 278–284.

[48] *Minimal size of a graph with diameter 2 and given maximal degree II.*, Acta Math. Univ. Comen. **61** (1992), 209–217.

[49] *On the spectrum for maximal partial Steiner triple systems* (with C. J. Colbourn and A. Rosa), Designs, Codes and Cryptography **3** (1993), 209–219.

[50] *Packing pentagons into complete graphs: how clumsy can you get* (with A. Rosa), Discrete Math. (To appear).

[51] *Digraphs of degree 3 close to Moore bound* (with E. T. Baskoro, M. Miller and J. Plesník). (Submitted).

[52] *Regular digraphs of diameter 2 and maximum order* (with E. T. Baskoro, M. Miller and J. Plesník). (Submitted).

B. Other papers

[53] *A research into the potential of high school graduates for studying mathematics in technical universities* (Slovak, co-author), Matematika ve škole **12** (1962-63), 273–279.

[54] *Selected topics in elementary number theory* (Slovak), Mat. obzory **3**, **4**, 37–56, 47–64.

[55] *Playing with Mathematics* (Hungarian), Természet és társadalom **11** (1966).

[56] *Mr. Teacher* (Slovak), Pokroky MFA **3** (1971).

[57] *Exempla trahunt...* (Slovak), Mat. obzory **1**, 9–17.

[58] *Review of A. Rényi's book "Ars Mathematica"* (Slovak), Mat. obzory **6**, 87–88.

[59] *Computer solves the four colour problem* (Slovak), Elektrón **3** (1977).

[60] *Can you draw it without lifting the pencil from the paper?* (Slovak), Elektrón **9** (1977).

[61] *On certain philosophical questions of mathematics* (Slovak, co-author). In: Facultatis Pedagogica Tyrnaviensis, Mathematica I., pp. 77–82.

[62] *Miracle tables* (Slovak), Priateľ **10** (1982), 5.

[63] *The seven bridges* (Slovak), Priateľ **11** (1982), 4.

[64] *How to colour maps* (Slovak), Priateľ **2** (1983), 5.

[65] *Who says* 100 (Slovak), Priateľ **5** (1983), 4.

[66] *Rabbits and the golden section* (Slovak), Elektrón (1984).

Textbooks

[67] *Mathematics Textbook for Further Education of Elementary School Teachers* (Slovak, co-author), Comenius Univ., Bratislava, 1970, 1971 and 1974.

[68] *Linear Algebra* (Slovak), Comenius Univ., 1972 and 1975.

[69] *Combinatorics – Supplementary Textbook for Mathematics Lecturers* (Slovak, co-author), SPN, Bratislava, 1973.

[70] *Selected Topics in Mathematics* (Slovak, co-author), Comenius Univ., Bratislava, 1974.

[71] *Combinatorics – Textbook for Grade 6 – 9 Mathematics Teachers* (Slovak, co-author), SPN, Bratislava, 1974.

[72] *Number Theory – Textbook for Grade 6 – 9 Mathematics Teachers* (Slovak, co-author), SPN, Bratislava, 1974.

[73] *Combinatorics and Graph Theory* (Slovak), Comenius Univ., Bratislava, 1978, 1981 and 1982.

[74] *Introduction to Linear Algebra* (Slovak, co-author), Comenius Univ, Bratislava, 1979 and 1981.

[75] *Teaching Number Theory, Graph Theory and Combinatorics in Universities* (Slovak), In: Didactical and Methodological Aspects of Teaching Mathematics in Universities (T. Šalát et al.), URVŠ SSR, 1986.

[76] *Graph Theory, Combinatorics* (Slovak), In: Suggestions for Further Education of Teachers (T. Šalát et al.), Comenius Univ., Bratislava, 1988.

[77] *Combinatorics and Graph Theory I* (Slovak), Comenius Univ., Bratislava, 1989.

Monographs and books

[78] *Number Theory – Basic Concepts and a Collection of Problems* (Slovak, with U. S. Davydov), SPN, Bratislava, 1966 and 1972.

[79] *Number Theory* (Slovak), ALFA, Bratislava, 1976 and 1985.

[80] *Mathematics for High School* (Slovak, with J. Šedivý, L. Kosmák and J. Smíd), SPN, Bratislava, 1978 and 1979.

[81] *Father, Mother, Mathematics and I* (Slovak, with B. Riečan), SPN, Bratislava, 1982.

[82] *A look into the history of mathematics* (Slovak, with L. Bukovský, M. Hejný, J. Hvorecký and B. Riečan), ALFA, Bratislava, 1986, (Croatian translation appeared in: Knjiga, Zagreb 1989).

this seminar after Bosák's death in 1987; this seminar has become one of the most productive seminars in Slovakia. In addition to many administrative duties, Š t e f a n Z n á m served as editor-in-chief of the journal Acta Mathematica Universitatis Comenianae.

Š t e f a n Z n á m spent a term in 1984 at the University of Waterloo, Canada, and again in 1991 at McMaster University in Hamilton, Canada. As recently as in spring of 1993, he visited the University of Newcastle in Australia.

The sudden passing of Professor Š t e f a n Z n á m is a great loss for his friends, students, and the whole mathematical community. We shall remember him for his friendliness and devotion, and, at the same time, high standards that he demanded of himself, his students and his surroundings. We shall miss his enthusiasm for mathematics with which he managed to attract so many to our beloved science.

Ján Plesník,
Štefan Porubský,
Alex Rosa,
Jozef Širáň

LIST OF PUBLICATIONS OF ŠTEFAN ZNÁM

REFERENCES

A. Scientific papers

[1] *A remark on a property of the two-element field* (Slovak), Mat.-Fyz. Časopis **11** (1961), 159–160.

[2] *A remark on a theorem of Eneström-Kakeya* (Slovak), Acta Facultatis R.N.U.C. Mathematica **7** (1963), 623–627.

[3] *Remark on an unpublished result of Pál Turán* (Hungarian), Mathematikai Lapok **14** (1963), 307–310.

[4] *On a combinatorial problem of K. Zarankiewicz*, Colloq. Math. **11** (1963), 81–84.

[5] *A note on a paper of J. Sedláček* (Slovak), Mat.-Fyz. Časopis **14** (1964), 263–264.

[6] *Two improvements of a result concerning a problem of K. Zarankiewicz*, Colloq. Math. **13** (1965), 255–258.

[7] *On a combinatorial problem in theory of congruences* (Russian; with A. Rosa), Mat.-Fyz. Časopis **15** (1965), 49–59.

[8] *A note on a combinatorial problem* (Russian; with A. Rosa), Mat.-Fyz. Časopis **15** (1965), 313–316.

[9] *Generalization of a number-theoretic result*, Mat.-Fyz. Časopis **16** (1966), 357–361.

[10] *On a Mycielski's problem on systems of arithmetical progressions*, Colloq. Math. **15** (1966), 201–204.

[11] *On symmetric and cyclic mean of positive numbers* (Slovak; with P. Bartoš), Mat.-Fyz. Časopis **16** (1966), 291–298.

[12] *On a decomposition of complete graphs into factors with given diameters* (with J. Bosák and A. Rosa). In: Proc. Colloq. Graph Theory, Tihany, 1966, pp. 37–56.

[13] *Modification of semi-completeness for integer sequences* (with J. Bosák and A. Kotzig), Mat.-Fyz. Časopis **16** (1966), 187–192.

[14] *On k-thin sets and n-extensive graphs*, Mat. Časopis **17** (1967), 297–307.

[15] *Equivalence of a number-theoretical problem with a problem from the graph theory*, Mat. Časopis **17** (1967), 238–239.

[16] *On some metric problems in graph theory* (with J. Bosák and A. Kotzig). In: Beiträge zur Graphentheorie, Manebach 1967, pp. 33–36.

[17] *On exactly covering systems of arithmetic sequences.* In: Colloq. Math. Soc. J. Bólyai 2. Number Theory. Debrecen 1968, pp. 221–224.

[18] *On sums of prime powers* (with T. Šalát), Acta Facultatis R.N.U.C. Mathematica **21** (1968), 21–24.

[19] *Strongly geodetic graphs* (with J. Bosák and A. Kotzig), J. Combin. Theory **5** (1968), 170–176.

[20] *A remark to a problem of J. Mycielski on arithmetic sequences*, Colloq. Math. **20** (1969), 69–70.

[21] *On exactly covering systems of arithmetic sequences*, Math. Ann. **180** (1969), 227–232.

[22] *A problem of Zarankiewicz* (with R.K.Guy). In: Recent Progress in Combinatorics, Acad. Press, 1969, pp. 237–245.

[23] *Decomposition of complete directed graphs into factors with given diameters.* In: Combinatorial Structures and their Applications, Calgary 1969, pp. 489–490.

[24] *On the average order of an arithmetical function* (with T. Šalát), Mat. Časopis **20** (1970), 233–238, (Errata ibid., p. 280).

[25] *Decomposition of the complete directed graph into two factors with given diameters*, Mat. Časopis **20** (1970), 254–256.

[26] *The minimal number of edges of a directed graph with given diameter*, Acta Facultatis R.N.U.C. Mathematica **24** (1970), 181–185.

[27] *Coset decomposition of Abelian groups* (with M. Hejný), Acta Facultatis R.N.U.C. Mathematica **25** (1971), 15–19.

[28] *On covering of rings by their residue classes* (with Š. Porubský), Acta Facultatis R.N.U.C. Mathematica **27** (1972), 45–51.

[29] *On decomposition of complete graphs into factors with given radii* (with D. Palumbíny), Mat. Časopis **23** (1973), 306–316.

[30] *Strongly geodetic directed graphs* (with J. Plesník), Acta Facultatis R.N.U.C. Mathematica **29** (1974), 29–34.

[31] *Disjoint covering systems* (with B. Novák), Amer. Math. Monthly **81** (1974), 42–45.

[32] *Vector-covering systems of arithmetic sequences*, Czechoslovak Math. J. **24** (1974), 455–461.

[33] *On covering sets of residue classes.* In: Colloq. Math. Soc. J. Bólyai 13. Number Theory Debrecen 1974, pp. 443–449.

[34] *On properties of systems of arithmetic sequences*, Acta Arithm. **26** (1975), 279–283

[35] *A simple characterization of disjoint covering systems*, Discrete Math. **12** (1975), 89–91

[36] *On existence and regularity of graphs with certain properties*, Discrete Math. **16** (1976), 83–90

[37] *On disjoint covering of groups by their cosets* (with I. Korec), Math. Slovaca **27** (1977), 3–7.